

Tensors, Quantum Entanglement & Optimization

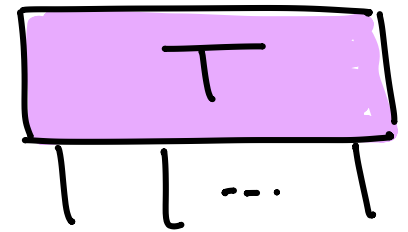
Michael Walter (LMU)

Budapest, July 2026



Tensors as Quantum States

$$|T\rangle \in \mathbb{C}^{n_1} \otimes \dots \otimes \mathbb{C}^{n_d}$$

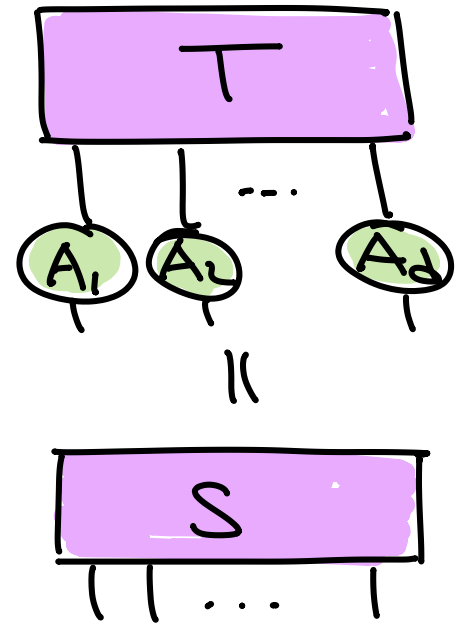


Tensors as Quantum States

$$|T\rangle \in \mathbb{C}^{n_1} \otimes \dots \otimes \mathbb{C}^{n_d}$$

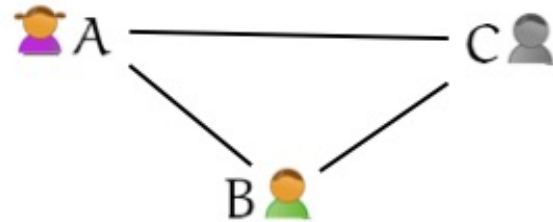
"SLOCC", "filtering",
"restriction"

$$S = (A_1 \otimes \dots \otimes A_d) T$$



Natural operations that "do not create entanglement"

tensor factors
 $\cong$
q. subsystems/
particles/parties/...

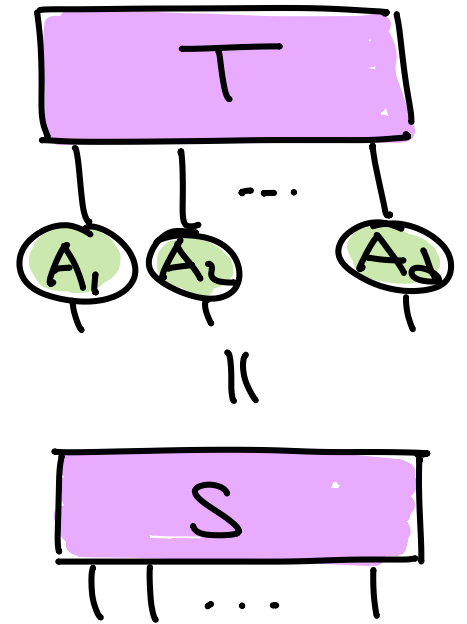


Tensors as Quantum States

$$|T\rangle \in \mathbb{C}^{n_1} \otimes \dots \otimes \mathbb{C}^{n_d}$$

"SLOCC", "filtering",
"restriction"

$$S = (A_1 \otimes \dots \otimes A_d) T$$



Natural operations that "do not create entanglement"

Can also allow limits: $S \in \overline{G \cdot T}$ "degeneration"

Where $G = GL(n_1) \times \dots \times GL(n_d)$

Q: Normal forms? Conversion (rules)? Algos?

Normal Forms of Quantum States

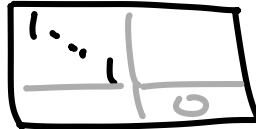
$$G = GL(n_1) \times \dots \times GL(n_d)$$

$d=2$: Gaussian elim. $\rightarrow$  $\hat{=}$ Singular values $(\underbrace{1, \dots, 1}_{\text{rank}})$

$d > 2$?

Normal Forms of Quantum States

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$d=2$: Gaussian elim. $\rightarrow$  $\hat{=}$ Singular values $(\underbrace{1, \dots, 1}_{\text{rank}})$

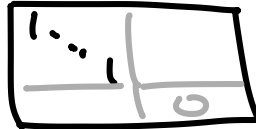
$d > 2$? Consider quantum marginals

$$Q_j = \begin{array}{|c|} \hline S \\ \hline \dots \\ \hline S' \\ \hline \end{array} = S_j S_j^* \text{ where } S_j : \mathbb{C}^{n_j} \times \dots$$

j^{th} principal flattening

Normal Forms of Quantum States

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$d=2$: Gaussian elim. $\rightarrow$  $\hat{=}$ Singular values $(\underbrace{1, \dots, 1}_{\text{rank}})$

$d > 2$? Consider quantum marginals

$$\sigma_j = \begin{array}{|c|} \hline S \\ \hline \dots \\ \hline S' \\ \hline \end{array} = S_j S_j^* \text{ where } S_j : \mathbb{C}^{n_j} \times \dots$$

j^{th} principal flattening

Proposal (Verstraëte et al, 2001):

- $S \in \overline{G \cdot T}$ is in normal form if $\sigma_j = I \ (\forall j)$
unique up to $K := U(n_1) \times \dots \times U(n_d)$

Normal Forms of Quantum States

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$d=2$: Gaussian elim. $\rightarrow$  $\hat{=}$ Singular values $(\underbrace{1, \dots, 1}_{\text{rank}})$

$d > 2$? Consider quantum marginals

$$\sigma_j = \begin{array}{|c|} \hline S \\ \hline \dots \\ \hline S^* \\ \hline \end{array} = S_j S_j^* \text{ where } S_j : \mathbb{C}^{n_j} \times \dots$$

j^{th} principal flattening

Proposal (Verstraëte et al, 2001):

- $S \in \overline{G \cdot T}$ is in normal form if $\sigma_j = I$ ($\forall j$)
unique up to $K := U(n_1) \times \dots \times U(n_d)$

• Algo:

$$S \leftarrow (I \otimes \dots \otimes I \otimes \sigma_j^{-1/2} \otimes I \otimes \dots \otimes I) S \quad (\forall j)$$

"Flip fbp", "tensor scaling"

Normal Forms of Quantum States (cont.) $G = G_L(n_1) \times \dots \times G_L(n_d)$

$$S \leftarrow (I \otimes \dots \otimes I \otimes \sigma_j^{-1/2} \otimes I \otimes \dots \otimes I) S$$

Analysis (Burgisser - ... - W-Widgerson, 2016): To find tensor
s.t. all $\sigma_j \approx_\epsilon I$, $\text{poly}(\text{input size}, \frac{1}{\epsilon})$ iterations suffice.

Normal Forms of Quantum States (cont.) $G = G_L(n_1) \times \dots \times G_L(n_d)$

$$S \longleftarrow (I \otimes \dots \otimes I \otimes \sigma_j^{-1/2} \otimes I \otimes \dots \otimes I) S$$

Analysis (Burgisser - ... - W-Widgerson, 2016): To find tensor s.t. all $\sigma_j \approx_\epsilon I$, $\text{poly}(\text{input size}, \frac{1}{\epsilon})$ iterations suffice.

key observation: iterations decrease

$$F(g_1, \dots, g_d) = \left\| \left(\bigotimes_{\substack{\det(g_j) = 1}} \det(g_j)^{-1/n_j} g_j \right) \cdot T \right\|^2$$

... and $\inf F > 0$ if T semistable

Normal Forms of Quantum States (cont.) $G = G_L(n_1) \times \dots \times G_L(n_d)$

$$S \longleftarrow (I \otimes \dots \otimes I \otimes \sigma_j^{-1/2} \otimes I \otimes \dots \otimes I) S$$

Analysis (Burgisser - ... - W-Widgerson, 2016): To find tensor s.t. all $\sigma_j \approx_\epsilon I$, $\text{poly}(\text{input size}, \frac{1}{\epsilon})$ iterations suffice.

key observation: iterations decrease

$$F(g_1, \dots, g_d) = \left\| \left(\bigotimes \underbrace{\det(g_j)^{-1/n_j}}_{\det = 1} g_j \right) \cdot T \right\|^2$$

AM/GM inequality!

... and $\inf F > 0$ if T semistable

make quantitative using invariant theory!

$$\begin{aligned} F_{\text{new}} &= \det(\sigma_j)^{1/n_j \cdot n_j} \\ &\leq \text{tr}(\sigma_j) = F_{\text{old}} \end{aligned}$$

Normal Forms of Quantum States (cont.) $G = G_L(n_1) \times \dots \times G_L(n_d)$

$$S \longleftarrow (I \otimes \dots \otimes I \otimes \sigma_j^{-1/2} \otimes I \otimes \dots \otimes I) S$$

Analysis (Bürgisser - ... - W-Wigderson, 2006): To find tensor s.t. all $\sigma_j \approx_\epsilon I$, $\text{poly}(\text{input size}, \frac{1}{\epsilon})$ iterations suffice.

key observation: iterations decrease

$$F(g_1, \dots, g_d) = \left\| \left(\bigotimes_{\substack{\det(g_j) = 1}} \det(g_j)^{-1/n_j} g_j \right) \cdot T \right\|^2$$

AM/GM inequality!

... and $\inf F > 0$ if T semistable

generic tensors

have normal form ∞

Same algo tests

semistability

Tensors & Norm Minimization

Q: Can we similarly optimize norms for other actions?
(And is it good for anything?) **A:** YES oo

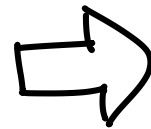
Tensors & Norm Minimization

Q: Can we similarly optimize norms for other actions?
(And is it good for anything?) **A:** YES oo

① Tensors as Data in Statistics:

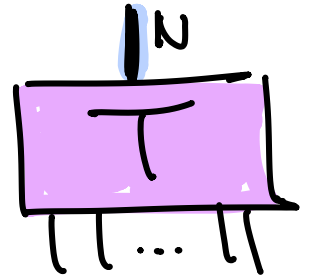
$$T_1, T_2, \dots \in \mathbb{R}^{n_1 \times \dots \times n_d}$$

Samples



$$T = (T_j)$$

$$\mathbb{R}^{n_1} \otimes \dots \otimes \mathbb{R}^{n_d} \otimes \mathbb{R}^N$$



Tensors & Norm Minimization

Q: Can we similarly optimize norms for other actions?
(And is it good for anything?) **A:** YES ∞

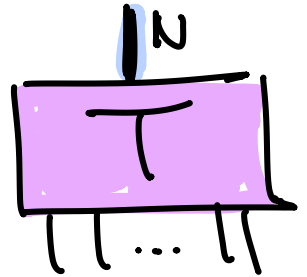
① Tensors as Data in Statistics:

$$T_1, T_2, \dots \in \mathbb{R}^{n_1 \times \dots \times n_d}$$

Samples

$$\Rightarrow T = (T_j)$$

$$\mathbb{R}^{n_1} \otimes \dots \otimes \mathbb{R}^{n_d} \otimes \mathbb{R}^N$$



Easy to see:

max likelihood in
"tensor normal model"
 $N(0, \Sigma_1 \otimes \dots \otimes \Sigma_d)$

$$\approx \min. \text{norm} \quad \|(G_1 \otimes \dots \otimes S_d \otimes I) T\|^2$$

Tensors & Norm Minimization

Q: Can we similarly optimize norms for other actions?
(And is it good for anything?) **A:** YES ∞

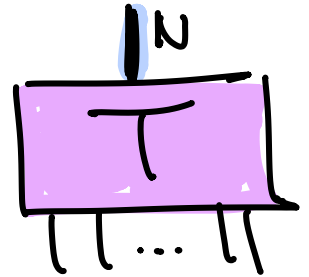
① Tensors as Data in Statistics:

$$T_1, T_2, \dots \in \mathbb{R}^{n_1 \times \dots \times n_d}$$

Samples

$$\Rightarrow T = (T_j)$$

$$\mathbb{R}^{n_1} \otimes \dots \otimes \mathbb{R}^{n_d} \otimes \mathbb{R}^N$$



Easy to see:

max likelihood in
"tensor normal model"
 $N(0, \Sigma_1 \otimes \dots \otimes \Sigma_d)$

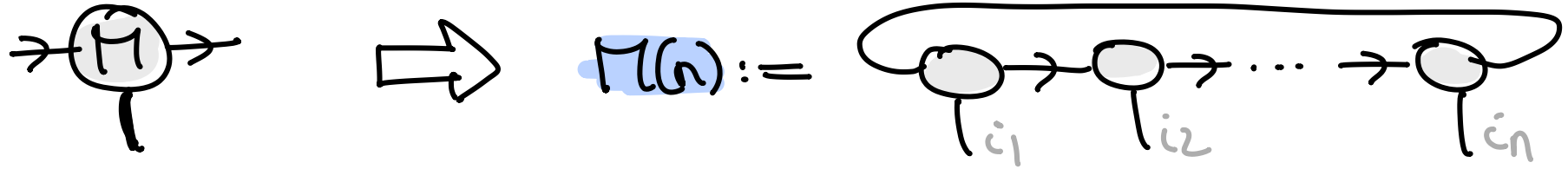
min. norm
 $\approx \|(G_1 \otimes \dots \otimes G_d \otimes I) T\|^2$

↳ "Same" flip-flop algo
Computes MLE ∞

see Améndola et al,
Franks - ... W

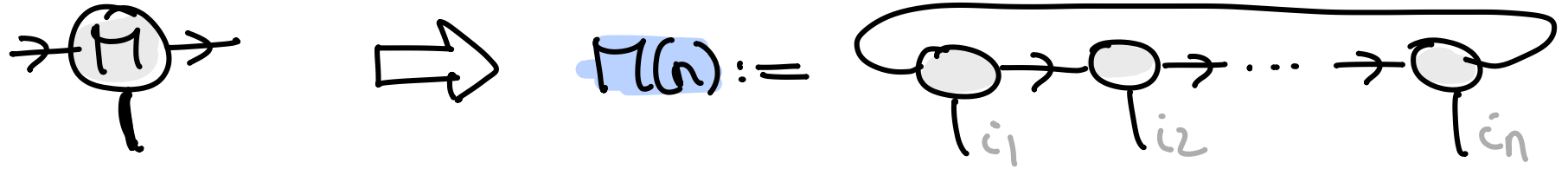
② Tensors define Tensor Networks:

(Uniform) Matrix Product States:



② Tensors Define Tensor Networks:

(Uniform) Matrix Product States:



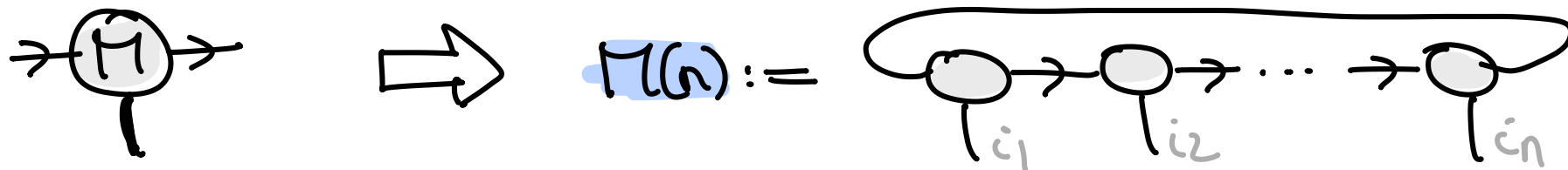
Invariant polys for "Simultaneous Conjugation":

$$GL(n) \hookrightarrow (\mathbb{C}^{n \times n})^{\oplus N}, \quad g \bullet (M_j) = (g M_j g^{-1})$$

(new) Canonical form by
minimizing $\|g \bullet M\|^2$
+ fundamental theorem

② Tensors define Tensor Networks:

(Uniform) Matrix Product States:



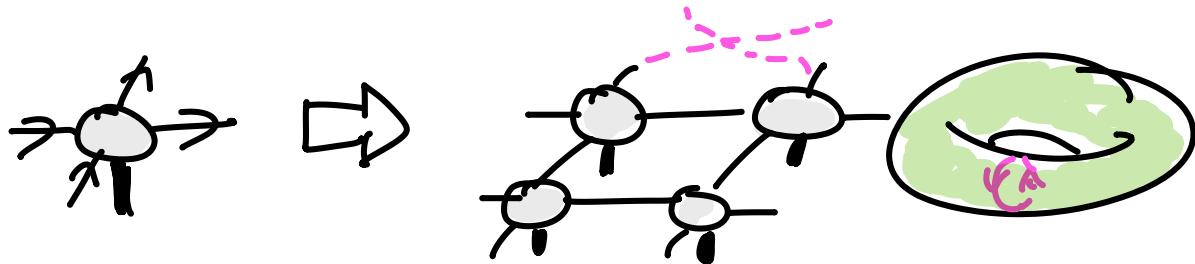
Invariant polys for "Simultaneous Conjugation":

$$GL(n) \subset (\mathbb{C}^{n \times n})^{\oplus N}, \quad g \bullet (M_j) = (g M_j g^{-1})$$



(new) Canonical form by
minimizing $\|g \bullet M\|^2$
+ fundamental theorem

→ generalizes to high dimensions
(but with a twist) PETS

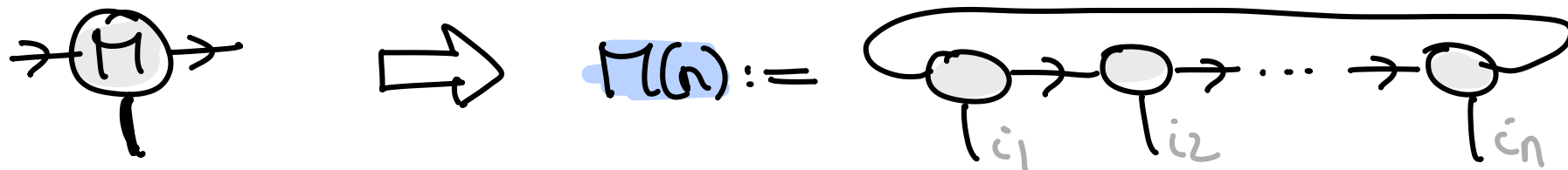


see Acuniva-...-W

5/20

② Tensors define Tensor Networks:

(Uniform) Matrix Product States:



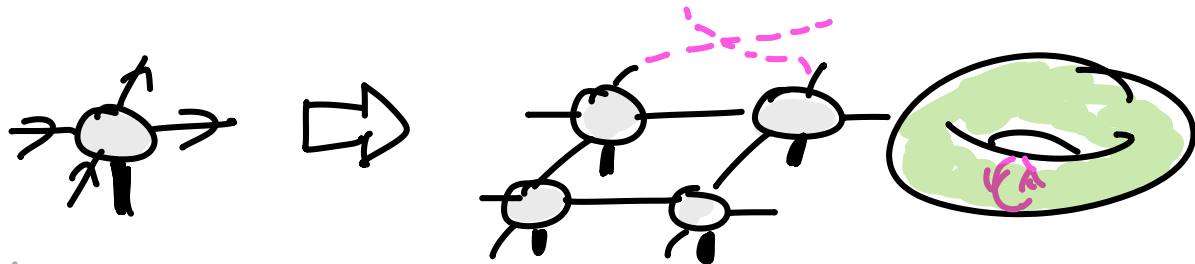
Invariant polys for "Simultaneous conjugation":

$$GL(n) \hookrightarrow (\mathbb{C}^{n \times n})^{\oplus N}, \quad g \bullet (M_j) = (g M_j g^{-1})$$



(new) Canonical form by
minimizing $\|g \bullet M\|^2$
+ fundamental theorem

→ generalizes to high dimensions
(but with a twist) PETS



② Semistability testing
& GIT $\cong$ symplectic quotient...

see Aravira-...-W

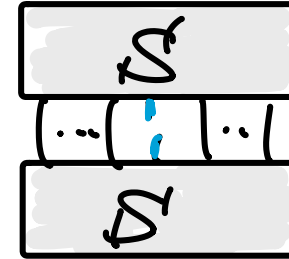
5/20

Back to Quantum States & Marginals

Recall: • Tensor action: $S = (g_1 \otimes \dots \otimes g_d)^T$

• Quantum marginals

$$Q_j =$$



Q: Which marginals can be achieved?

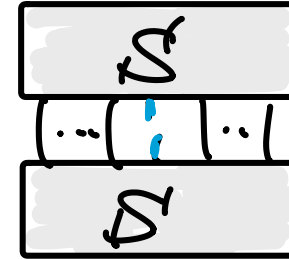
Normal forms & invariants also for unstable T ?

Back to Quantum States & Marginals

Recall: • Tensor action: $S = (g_1 \otimes \dots \otimes g_d)^T$

• Quantum marginals

$$\sigma_j =$$



Q: Which marginals can be achieved?

Normal forms & invariants also for unstable T ?

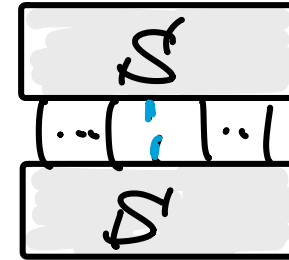
$$\{(\sigma_1, \dots, \sigma_d) : S \in \overline{G \cdot T}, \|S\| = 1\} = ?$$

Back to Quantum States & Marginals

Recall: • Tensor action: $S = (g_1 \otimes \dots \otimes g_d)^T$

• Quantum marginals

$$\sigma_j =$$



Q: Which marginals can be achieved?

Normal forms & invariants also for unstable T ?



Entanglement Polytope:

(W...-Christandl, 2002)

$$\Delta(T) = \{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\| = 1 \}$$

Wild but true: It's a polytope!

"hard" proof
(Mumford, Kirwan)

6/20

Entanglement Polytopes

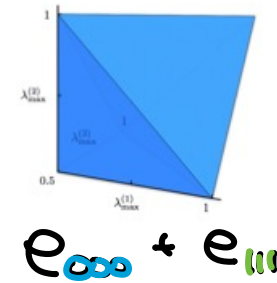
$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \left\{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \right\}$$

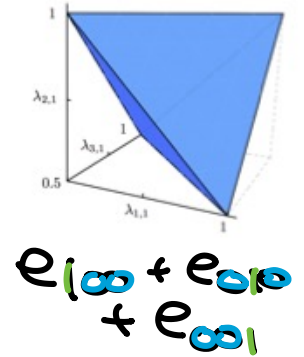
← prob. dist. →

Wild but true:
It's a polytope!

E.g. $n_1 = n_2 = n_3 = 2$:



vs



Entanglement Polytopes

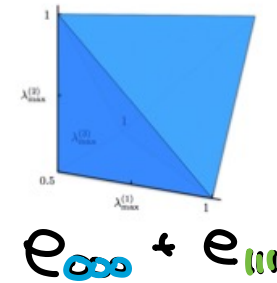
$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \left\{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \right\}$$

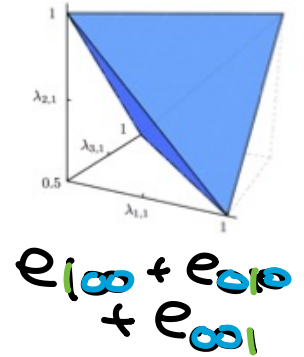
↙
prob. dist.
↘

Wild but true:
It's a polytope!

E.g. $n_1 = n_2 = n_3 = 2$:



vs



↑
even for T generic this is NOT trivial!
(a "quantum marginal problem")

Entanglement Polytopes

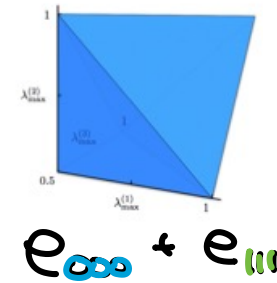
$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \left\{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \right\}$$

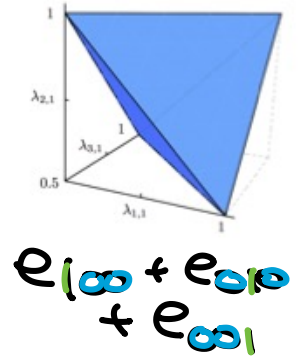
↙
↘
 prob. dist.

Wild but true:
It's a polytope!

E.g. $n_1 = n_2 = n_3 = 2$:



vs



↑
even for T generic this is NOT trivial!
(a "quantum marginal problem")

Properties: • $(\frac{1}{n_1}, \dots, \frac{1}{n_d}) \in \Delta(T) \iff$ semistable

has Vestrak et al
normal form

Entanglement Polytopes

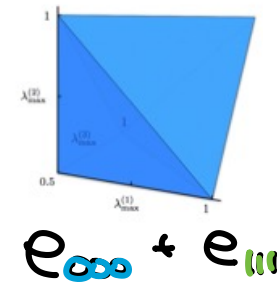
$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \left\{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \right\}$$

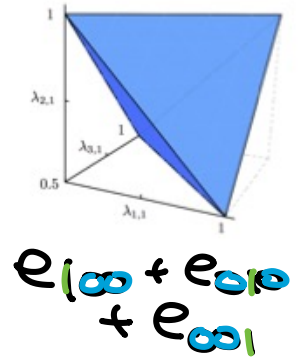
prob. dist.

Wild but true:
It's a polytope!

E.g. $n_1 = n_2 = n_3 = 2$:



vs



↑
even for T generic this is NOT trivial!
(a "quantum marginal problem")

Properties: • $(\frac{1}{n_1}, \dots, \frac{1}{n_d}) \in \Delta(T) \iff$ semistable $\searrow$

• Invariant: $\Delta(T) = \Delta(S \cdot T)$

has Verstraëke et al
normal form

• Monotonic: $S \in \overline{G \cdot T} \implies \Delta(S) \subseteq \Delta(T)$

↳ get "entanglement measures" by max. over $\Delta(T)$! 7/20

Entanglement Polytopes

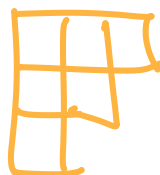
$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \}$$

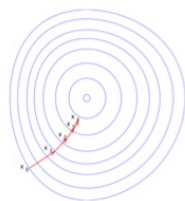
prob. dist.
 ↙ ↘

... have different descriptions, connecting (symplectic) geometry to

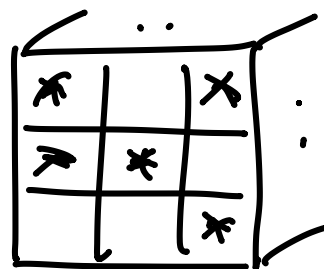
- representation theory



- optimization



- combinatorics of supports



Entanglement Polytopes

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \}$$

↙ prob. dist. ↘

... have different descriptions, connecting (symplectic) geometry to

- representation theory:

$$\Delta(T) = \{ (\frac{\lambda_1}{k}, \dots, \frac{\lambda_d}{k}) : (P_{\lambda_1} \otimes \dots \otimes P_{\lambda_d}) T^{\otimes k} \neq 0 \}$$

highest weights ↘
isotypical projections ↘
↖ > 0

- optimization
- combinatorics of supports

Entanglement Polytopes

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \left\{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \right\}$$

↙
prob. dist.
↘

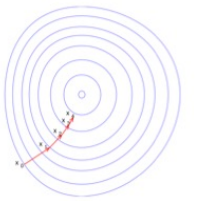
... have different descriptions, connecting (symplectic) geometry to

- representation theory:

$$\Delta(T) = \left\{ \left(\frac{\lambda_1}{k}, \dots, \frac{\lambda_d}{k} \right) : (P_{\lambda_1} \otimes \dots \otimes P_{\lambda_d}) T^{\otimes k} \neq 0 \right\}$$

↙
highest weights
↘
isotypical projections
↖
↖

- optimization: already saw "flip flop", more later...



- Combinatorics of supports

Entanglement Polytopes

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\overline{\Delta(T)} = \{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \}$$

↙ prob. dist. ↘

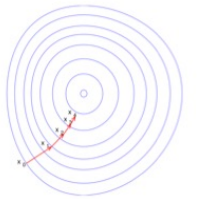
... have different descriptions, connecting (symplectic) geometry to

- representation theory:

$$\Delta(T) = \{ (\frac{\lambda_1}{k}, \dots, \frac{\lambda_d}{k}) : (P_{\lambda_1} \otimes \dots \otimes P_{\lambda_d}) T^{\otimes k} \neq 0 \}$$

highest weights ↘
isotypical projections ↘
↖ > 0

- optimization: already saw "flip flop", more later...



- Combinatorics of supports:

$$\Omega(S) = \text{Conv} \left\{ (e_{i_1}, \dots, e_{i_d}) \mid \begin{array}{l} \text{s.t.} \\ S_{i_1 \dots i_d} \neq 0 \end{array} \right\}$$

Support
polytope



Entanglement Polytopes

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\overline{\Delta(T)} = \{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \}$$

prob. dist.

... have different descriptions, connecting (symplectic) geometry to

- representation theory:

$$\Delta(T)'' = \{ (\frac{\lambda_1}{k}, \dots, \frac{\lambda_d}{k}) : (P_{\lambda_1} \otimes \dots \otimes P_{\lambda_d}) T^{\otimes k} \neq 0 \}$$

highest weights isotypical projections

- optimization: already saw "flip flop", more later...

- Combinatorics of supports:

$$\Omega(S') = \text{Conv} \left\{ (e_{i_1}, \dots, e_{i_d}) \mid \substack{\text{s.t.} \\ S_{i_1 \dots i_d} \neq 0} \right\} \Rightarrow \Delta(T)'' = \bigcap_{b \in B_-} \Omega(b \cdot g \cdot T)$$

generic

lower triangular

8/20

Support polytope

Entanglement Polytopes

$$G = GL(n_1) \times \dots \times GL(n_d)$$

$$\boxed{\Delta(T)} = \{ (\text{spec } \sigma_1, \dots, \text{spec } \sigma_d) : S \in \overline{G \cdot T}, \|S\|=1 \}$$

prob. dist.

... have different descriptions, connecting (symplectic) geometry to

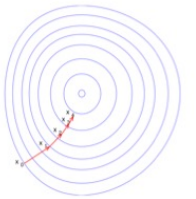
- representation theory:

↳ talk by Maxim

$$\Delta(T) = \{ (\frac{\lambda_1}{k}, \dots, \frac{\lambda_d}{k}) : (P_{\lambda_1} \otimes \dots \otimes P_{\lambda_d}) T^{\otimes k} \neq 0 \}$$

highest weights isotypical projections

- optimization: already saw "flip flop", more later...



- Combinatorics of supports:

$$\Omega(S) = \text{Conv} \left\{ (e_{i_1}, \dots, e_{i_d}) \mid \substack{\text{s.t.} \\ S_{i_1 \dots i_d} \neq 0} \right\} \Rightarrow \Delta(T) = \bigcap_{b \in B_-} \Omega(b \cdot g \cdot T)$$

generic

Support
polytope

Similarly for any complex reductive G . lower triangular 8/20

Tensors as Computational Problems

multiplication of
r numbers

$$\triangle_r = \sum_{j=1}^r e_j \otimes e_j \otimes e_j$$

vs

$n \times n$ matrix multiplication

$$\begin{array}{c} n \quad n \\ \diagdown \quad \diagup \\ n \end{array} = \sum_{i,j,k=1}^n e_{ij} \otimes e_{jk} \otimes e_{ik}$$

Tensors as Computational Problems

multiplication of
r numbers

$$\triangle_r = \sum_{j=1}^r e_j \otimes e_j \otimes e_j$$

vs

$n \times n$ matrix multiplication

$$\begin{array}{c} n \quad n \\ \diagdown \quad \diagup \\ n \end{array} = \sum_{i,j,k=1}^n e_{ij} \otimes e_{jk} \otimes e_{ik}$$

reduction between problems $\hat{=}$ restriction/degeneration of tensors

Tensors as Computational Problems

multiplication of
r numbers

$$\triangle_r = \sum_{j=1}^r e_j \otimes e_j \otimes e_j$$

vs

$n \times n$ matrix multiplication

$$\frac{\begin{array}{c} n \text{ (green)} \quad n \text{ (orange)} \\ \text{---} \\ n \text{ (blue)} \end{array}}{n} = \sum_{i,j,k=1}^n e_{ij} \otimes e_{jk} \otimes e_{ik}$$

reduction between problems $\hat{=}$ restriction/degeneration of tensors

$\hookrightarrow$ algebraic complexity of matrix multiplication captured by:

$$\omega := \lim_{n \rightarrow \infty} \min \left\{ \frac{\log r}{\log n} : \triangle_r \longrightarrow \frac{\begin{array}{c} n \text{ (green)} \quad n \text{ (orange)} \\ \text{---} \\ n \text{ (blue)} \end{array} \right\}$$

Interpretation:

$O(n^\omega)$ operations suffice to
multiply two $n \times n$ matrices

Tensors as Computational Problems

multiplication of
r numbers

$$\triangle_r = \sum_{j=1}^r e_j \otimes e_j \otimes e_j$$

vs

$n \times n$ matrix multiplication

$$\frac{\triangle_n}{n} = \sum_{i,j,k=1}^n e_{ij} \otimes e_{jk} \otimes e_{ik}$$

reduction between problems $\hat{=}$ restriction/degeneration of tensors

↳ algebraic complexity of matrix multiplication captured by:

$$\omega := \lim_{n \rightarrow \infty} \min \left\{ \frac{\log r}{\log n} : \triangle_r \longrightarrow \frac{\triangle_n}{n} \right\}$$

$$= \lim_{n \rightarrow \infty} \min \left\{ \frac{k}{\ell} : \triangle_2^{\otimes k} \longrightarrow \left(\frac{\triangle_2}{2} \right)^{\otimes \ell} \right\}$$

Interpretation:

$O(n^\omega)$ operations suffice to
multiply two $n \times n$ matrices

Tensor Parameters & Entanglement Polytope

Quantum Functionals (Oristanelli et al, 2017): For $\theta \in \mathbb{R}_{\geq 0}^d, \sum \theta_j = 1$

$$F_{\theta}(T) := 2^{E_{\theta}(T)} \quad \text{where} \quad E_{\theta}(T) := \max_{x \in \Delta(T)} \sum_{j=1}^d \theta_j H(x_j)$$

Entanglement
polytope

Shannon entropy:
 $H(p) = \sum_i p_i \cdot \log \frac{1}{p_i}$

Tensor Parameters & Entanglement Polytope

Quantum Functionals (Ostindl et al, 2017): For $\theta \in \mathbb{R}_{\geq 0}^d, \sum \theta_j = 1$

$$F_{\theta}(T) := 2^{E_{\theta}(T)} \quad \text{where} \quad E_{\theta}(T) := \max_{x \in \Delta(T)} \sum_{j=1}^d \theta_j H(x_j)$$

Entanglement
polytope

Shannon entropy:
 $H(p) = \sum_i p_i \cdot \log \frac{1}{p_i}$

$d=2$: $F_{\theta} \equiv \text{rank}$
 $d>2$: NOT tensor rank ∞

Tensor Parameters & Entanglement Polytope

Quantum Functionals (Christandl et al, 2017): For $\Theta \in \mathbb{R}_{\geq 0}^d, \Sigma = 1$

$$F_{\Theta}(T) := 2^{E_{\Theta}(T)} \quad \text{where} \quad E_{\Theta}(T) := \max_{X \in \Delta(T)} \sum_{j=1}^d \Theta_j H(x_j)$$

Entanglement
polytope

Shannon entropy:
 $H(p) = -\sum_i p_i \cdot \log \frac{1}{p_i}$

$d=2$: $F_{\Theta} \equiv \text{rank}$
 $d>2$: NOT tensor rank ∞

Wild, but true: Multiplicative w.r.t. $\otimes$, additive w.r.t. $\oplus$ ← "hard" proof
 (+ normalized + monotone)

↳ bounds asymptotic conversion rates:

$$T^{\otimes k} \rightarrow S^{\otimes \ell} \quad \Rightarrow \quad \frac{k}{\ell} \geq \frac{E_{\Theta}(S)}{E_{\Theta}(T)}$$

cf. talk
by Jeroen

$$\hookrightarrow \underline{\text{rank}}(T) \geq F_{\Theta}(T) \geq \underline{\text{subrank}}(T)$$

10/20

Tensor Parameters & Entanglement Polytope

Several recent **tensor parameters** (= entanglement measures) are defined by **convex/concave optimization** over **entanglement polytope** !

Tensor Parameters & Entanglement Polytope

Several recent **tensor parameters** (= entanglement measures) are defined by **convex/concave optimization** over **entanglement polytope** !

↓

Q. Functionals

$$E_{\Theta}(T) = \max_{x \in \Delta(T)} \sum_{j=1}^d \theta_j \overset{\text{entropy}}{H(x_j)}$$

Tensor Parameters & Entanglement Polytope

Several recent **tensor parameters** (= entanglement measures) are defined by **convex/concave optimization** over **entanglement polytope** !



Q. Functionals

$$E_{\Theta}(T) = \max_{x \in \Delta(T)} \sum_{j=1}^d \theta_j \overset{\text{entropy}}{H(x_j)}$$



Q-stable rank (DedSca)

$$rk_{\alpha}^Q(T) = \left[\min_{x \in \Delta(T)} \max_{j \in [d]} \frac{\|x_j\|_{\infty}}{\alpha_j} \right]^{-1}$$

Tensor Parameters & Entanglement Polytope

Several recent **tensor parameters** (= entanglement measures) are defined by **convex/concave optimization** over **entanglement polytope** !

Q. Functionals

$$E_{\Theta}(T) = \max_{x \in \Delta(T)} \sum_{j=1}^d \Theta_j \overset{\text{entropy}}{H(x_j)}$$

l^2 -norm also for Θ on action

Non-Commutative rank

$$\text{nck}(T) = n - \frac{n}{2} \min_{x \in \Delta(T)} \sum_{j=1}^2 \|p_j - \frac{1}{n}\|_1$$

G -stable rank (Dedekind)

$$\text{rk}_\alpha^G(T) = \left[\min_{x \in \Delta(T)} \max_{j \in [d]} \frac{\|x_j\|_\infty}{\alpha_j} \right]^{-1}$$

Tensor Parameters & Entanglement Polytope

Several recent tensor parameters (= entanglement measures) are defined by convex/concave optimization over entanglement polytope !

↓
Q. Functionals
entropy

↓
G-stable rank (Dedekind)
related to $\|\cdot\|_\infty$

↓
NC rank
related to $\|\cdot\|_1$

↓
 ℓ^2 -norm
also for $\mathfrak{g}$ on action

Results (Doğan-Sakabe-W, 2026):

- Efficient algorithms for all these: $\text{poly}(\frac{1}{\varepsilon}, \text{input size})$

Most interesting if T NOT generic (otherwise computes NF). 11/20

Tensor Parameters & Entanglement Polytope

Several recent **tensor parameters** (= entanglement measures) are defined by **convex/concave optimization** over **entanglement polytope** !

↓
Q. Functionals
entropy

↓
G-stable rank (Dekser)
related to $\|\cdot\|_\infty$

↓
NC rank
related to $\|\cdot\|_1$

↓
 ℓ^2 -norm
also for g on action

Results (Doğan-Sakabe-W, 2026):

- Efficient algorithms for all these: $\text{poly}(\frac{1}{\epsilon}, \text{input size})$

- For q. functionals, "entropic tensor scaling"

$$S \leftarrow (\sigma_1^{-\Theta/2} \otimes \dots \otimes \sigma_d^{-\Theta/2}) S$$

- Structural insights: simple proofs that T_Θ is $\otimes$ & $\oplus$

Most interesting if T **NOT** generic (otherwise computes NF). 11/20

Algos for Tensor Parameters from Ent. Polytopes

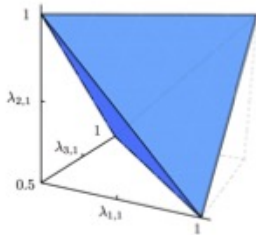
Results (Doğan-Sakabe-W, 2026):

- Efficient algorithms for all these: $\text{poly}(\frac{1}{\epsilon}, \text{input size})$
- For q . functionals, "entropic tensor scaling"

$$\mathcal{S} \mapsto (\sigma_1^{-\Theta/2} \otimes \dots \otimes \sigma_d^{-\Theta/2}) \mathcal{S}$$

- Structural insights: simple proofs that T_Θ is $\otimes$ & $\oplus$

e.g.



$$T = e_{\infty} + e_{\infty} + e_{\infty}$$

$$\hookrightarrow E_{(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})}(T) = H(\{\frac{2}{3}, \frac{1}{3}\})$$

Algos for Tensor Parameters from Ent. Polytopes

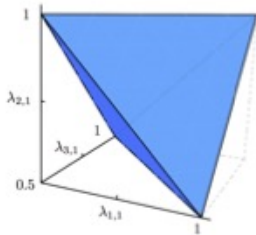
Results (Doğan-Sakabe-W, 2026):

- Efficient algorithms for all these: $\text{poly}(\frac{1}{\varepsilon}, \text{input size})$
- For q . functionals, "entropic tensor scaling"

$$\mathcal{S} \leftarrow (\sigma_1^{-\Theta_1/2} \otimes \dots \otimes \sigma_d^{-\Theta_d/2}) \mathcal{S}$$

- Structural insights: simple proofs that T_Θ is $\otimes$ & $\oplus$

e.g.



$$T = e_{\infty} + e_{\infty} + e_{\infty}$$

$$\hookrightarrow E_{(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})}(T) = H(\{\frac{2}{3}, \frac{1}{3}\})$$

Indeed:

$$\sigma_j \equiv \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \text{ \& hence } T \text{ "fixed" by algo!}$$

Algos for Tensor Parameters from Ent. Polytopes

Results (Doğan-Sakabe-W, 2026):

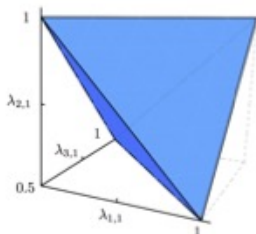
- Efficient algorithms for all these: $\text{poly}(\frac{1}{\varepsilon}, \text{input size})$

- For q . functionals, "entropic tensor scaling"

$$S \mapsto (\sigma_1^{-\Theta/2} \otimes \dots \otimes \sigma_d^{-\Theta/2}) S$$

- Structural insights: simple proofs that T_Θ is $\otimes$ & $\oplus$

e.g.



$$T = e_{\infty} + e_{\infty} + e_{\infty}$$

$$\hookrightarrow E_{(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})}(T) = H(\{\frac{2}{3}, \frac{1}{3}\})$$

Indeed:

$$\sigma_j \equiv \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \text{ \& hence } T \text{ fixed by algo!}$$

$$S = S' \otimes S'' \text{ preserved by algo} \\ \Rightarrow E_\Theta(S) \leq E_\Theta(S') + E_\Theta(S'')$$

Algos for Tensor Parameters from Ent. Polytopes

Results (Doğan-Sakabe-W, 2026):

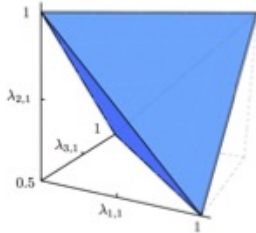
- Efficient algorithms for all these: $\text{poly}(\frac{1}{\epsilon}, \text{input size})$

- For q . functionals, "entropic tensor scaling"

$$S \mapsto (\sigma_1^{-\Theta/2} \otimes \dots \otimes \sigma_d^{-\Theta/2}) S$$

- Structural insights: simple proofs that T_Θ is $\otimes$ & $\oplus$

e.g.



$$T = e_{\infty} + e_{\infty} + e_{\infty}$$

$$\hookrightarrow E_{(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})}(T) = H(\{\frac{2}{3}, \frac{1}{3}\})$$

Indeed:

$$\sigma_j \equiv \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \text{ \& hence } T \text{ fixed by algo!}$$

$$S = S' \otimes S'' \text{ preserved by algo}$$

$$\Rightarrow E_\Theta(S) \leq E_\Theta(S') + E_\Theta(S'')$$

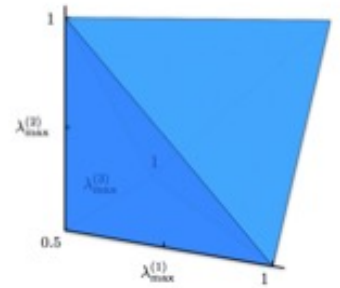
... and $\geq$ is easy

Similarly
for $\oplus$.

Optimization on Polytopes

Classical problem:

$$\left[\begin{array}{l} \min \phi(x) \\ x \in \triangle \\ \text{polytope} \end{array} \right] \quad \text{Convex}$$

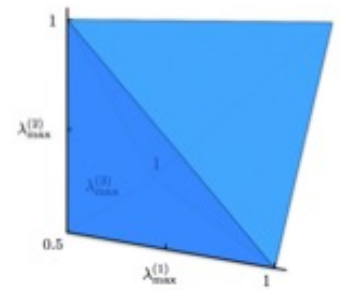


Optimization on Polytopes

Classical problem:

$$\left[\begin{array}{l} \min \phi(x) \\ x \in \triangle \\ \text{polytope} \end{array} \right] \quad \text{Convex}$$

"explicit"
if $\triangle$ given as vertices/facets:
Standard methods such as ellipsoid



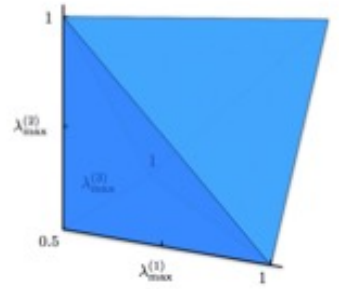
Optimization on Polytopes

Classical problem:

$$\boxed{\begin{array}{l} \min \phi(x) \\ x \in \triangle \\ \text{polytope} \end{array}} \quad \text{Convex}$$

if $\triangle$ implicitly defined,
Can sometimes still solve,
also if exponential # of
vertices & facets (e.g. matroid base polytope)

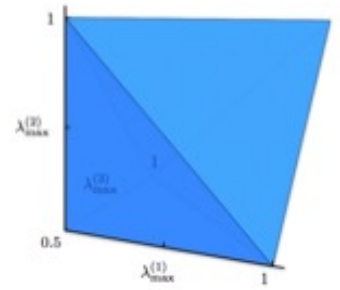
if $\triangle$ given as vertices/facets:
Standard methods such as ellipsoid



Optimization on Polytopes

Classical problem:

$$\boxed{\begin{array}{l} \min \phi(x) \\ x \in \triangle \\ \text{polytope} \end{array}} \quad \text{Convex}$$



if Δ implicitly defined,
Can sometimes still solve,
even if exponential # of
vertices & facets (e.g. matroid base polytope)

if Δ given as vertices/facets:
Standard methods such as ellipsoid

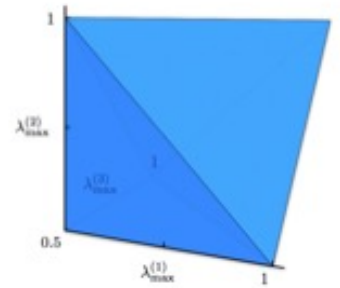
Example:

$$\triangle = \text{conv} \{ \omega_j \}$$

Optimization on Polytopes

Classical problem:

$$\boxed{\begin{array}{l} \min \phi(x) \\ x \in \triangle \\ \text{polytope} \end{array}} \quad \text{Convex}$$



if $\triangle$ implicitly defined,
Can sometimes still solve,
even if exponential # of
vertices & facets (e.g. matroid base polytope)

if $\triangle$ given as vertices/facets:
Standard methods such as ellipsoid

Example:

$$\triangle = \frac{\text{conv} \{ \omega_j \}}{= \{ \nabla f(x) \}}$$

for $f: \mathbb{R}^n \rightarrow \mathbb{R}$
 $f(x) = \log \left[\sum_j e^{x \cdot \omega_j} \right]$

↳ polytope succinctly encoded as "gradient set" of convex f ! 13/20

Polytopes from Group Actions

Kirwan, Ness,
... Brieskorn...-W

Can similarly associate polytopes to any "nice" group action?

E.g. entanglement polytopes:

$$G = GL(n_1) \times \dots \times GL(n_d) \hookrightarrow V = \mathbb{C}^{n_1} \otimes \dots \otimes \mathbb{C}^{n_d}$$

Polytopes from Group Actions

Kirwan, Ness,
... Buser...-W

Can similarly associate polytopes to any "nice" group action?

E.g. entanglement polytopes:

$$G = GL(n_1) \times \dots \times GL(n_d) \hookrightarrow V = \mathbb{C}^{n_1} \otimes \dots \otimes \mathbb{C}^{n_d}$$

To any $T \in V$, associate fn. on $P = PD(n_1) \times \dots \times PD(n_d)$:

$$f: P \rightarrow \mathbb{R}, \quad f(p) = \log \underbrace{\langle T, (p_1 \otimes \dots \otimes p_d) T \rangle}_{= \|\sqrt{p} \cdot T\|^2}$$

Polytopes from Group Actions

Kirwan, Ness,
... Brieskorn...-W

Can similarly associate polytopes to any "nice" group action?
E.g. entanglement polytopes:

$$G = GL(n_1) \times \dots \times GL(n_d) \hookrightarrow V = \mathbb{C}^{n_1} \otimes \dots \otimes \mathbb{C}^{n_d}$$

To any $T \in V$, associate fn. on $P = PD(n_1) \times \dots \times PD(n_d)$:

$$f: P \rightarrow \mathbb{R}, \quad f(p) = \log \langle T, (p_1 \otimes \dots \otimes p_d) T \rangle$$

$= \|\sqrt{p} \cdot T\|^2$

Easy to verify:

$$\nabla f(p) = \bar{p}^{1/2} (\sigma_1, \dots, \sigma_d) p^{-1/2}$$

where σ_j q. marginals of $S = \sqrt{p} \cdot T_{/1\dots 1}$ ☺

$$\hookrightarrow \Delta(T) = \overline{\{ \text{spec } \sqrt{p} \nabla f(p) \sqrt{p} : p \in P \}}$$

14/20

Entanglement Polytope from Group Action

Almost a "gradient set":

$$\Delta(T) = \overline{\{ \text{spec } \sqrt{p} \nabla f(p) \sqrt{p} : p \in P \}}$$

$$f: P \rightarrow \mathbb{R}$$
$$f(p) = \log \langle T, p \cdot T \rangle$$

$$P = PD(n_1) \times \dots \times PD(n_d)$$

Entanglement Polytope from Group Action

Almost a "gradient set":

$$\Delta(T) = \overline{\left\{ \text{spec } \sqrt{p} \nabla f(p) \sqrt{p} : p \in P \right\}}$$

$$f: P \rightarrow \mathbb{R}$$
$$f(p) = \log \langle T, p \cdot T \rangle$$

$$P = \text{PD}(n_1) \times \dots \times \text{PD}(n_d)$$

☹️ f is **NOT** convex
in Euclidean sense

Entanglement Polytope from Group Action

Almost a "gradient set":

$$\Delta(T) = \overline{\left\{ \text{Spec } \sqrt{p} \nabla f(p) \sqrt{p} : p \in P \right\}}$$

$$f: P \rightarrow \mathbb{R}$$

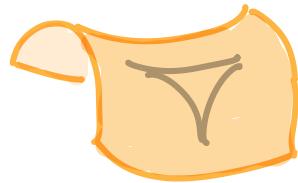
$$f(p) = \log \langle T, p \cdot T \rangle$$

$$P = PD(n_1) \times \dots \times PD(n_d)$$

☹️ f is **NOT** convex in Euclidean sense

😊 f is convex along geodesics of P as Riemannian manifold:

$$\gamma(t) = \sqrt{p} e^{Xt} \sqrt{p}$$



😊 $\sqrt{p} \cdot \sqrt{p}$ is parallel transport

Likewise for any G . If commutative: $P \cong \mathbb{R}^n$, $f \cong \log\text{-sum-exp}$. 15/20

Excursion: PD(N), Curvature, Convexity

$$\boxed{\text{PD}(N) \subseteq \text{Herm}(N)} \text{ open}$$

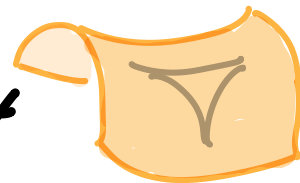


Euclidean metric:

$$\langle X, Y \rangle_P \equiv \text{tr}[XY]$$

- flat
- geodesics $\gamma(t) = p + tX$

Excursion: PD(N), Curvature, Convexity



$$\text{PD}(N) \subseteq \text{Herm}(N)$$

↙
Euclidean metric:

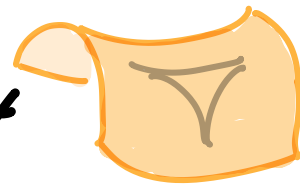
$$\langle X, Y \rangle_p \equiv \text{tr}[XY]$$

- flat
- geodesics $\gamma(t) = p + tX$

↘
"affine-invariant" metric

$$\langle X, Y \rangle_p = \text{tr}[p^{-1}Xp^{-1}Y]$$

Excursion: PD(N), Curvature, Convexity



$$\text{PD}(N) \subseteq \text{Herm}(N)$$

↙
Euclidean metric:

$$\langle X, Y \rangle_p \equiv \text{tr}[XY]$$

- flat
- geodesics $\gamma(t) = p + tX$

↘
"affine-invariant" metric

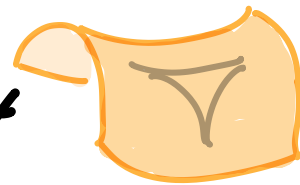
$$\langle X, Y \rangle_p = \text{tr}[p^{-1}Xp^{-1}Y]$$

- Curvature ≤ 0
- geodesics: $\gamma(t) = \sqrt{p} e^{Xt} \sqrt{p}$

Connecting uniquely any two pts

"Hadamard manifold"

Excursion: PD(N), Curvature, Convexity



$$\text{PD}(N) \subseteq \text{Herm}(N)$$

Euclidean metric:

$$\langle X, Y \rangle_p \equiv \text{tr}[XY]$$

- flat
- geodesics $\gamma(t) = p + tX$

"affine-invariant" metric

$$\langle X, Y \rangle_p = \text{tr}[p^{-1}Xp^{-1}Y]$$

- Curvature ≤ 0
- geodesics: $\gamma(t) = \sqrt{p}e^{Xt}\sqrt{p}$

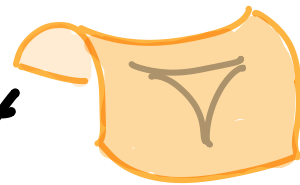
Connecting uniquely any two pts
"Hadamard manifold"

For any $v \in \mathbb{C}^N$:

$$f: \text{PD}(N) \rightarrow \mathbb{R}, \quad f(p) := \log \langle v, pv \rangle$$

is convex, i.e. $\partial_t^2 f(\gamma(t)) \geq 0$.

Excursion: PD(N), Curvature, Convexity



$$\text{PD}(N) \subseteq \text{Herm}(N)$$

Euclidean metric:

$$\langle X, Y \rangle_p \equiv \text{tr}[XY]$$

- flat
- geodesics $\gamma(t) = p + tX$

"affine-invariant" metric

$$\langle X, Y \rangle_p = \text{tr}[p^{-1}Xp^{-1}Y]$$

- Curvature ≤ 0
- geodesics: $\gamma(t) = \sqrt{p} e^{Xt} \sqrt{p}$

Connecting uniquely any two pts
"Hadamard manifold"

For any $v \in \mathbb{C}^N$:

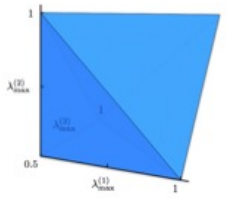
$$f: \text{PD}(N) \rightarrow \mathbb{R}, \quad f(p) := \log \langle v, pv \rangle$$

is convex, i.e. $\partial_t^2 f(\gamma(t)) \geq 0$.

→ talk to Keiya

Similarly for $\{p_1 \otimes \dots \otimes p_d\} \leftarrow$ convex subset!

Optimization on Entanglement Polytope



$$\min_{x \in \Delta(T)} \phi(x)$$

e.g. $\phi = \sum \text{Shannon entropy}$

Optimization on Entanglement Polytope

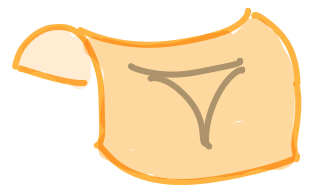
$$\min_{x \in \Delta(\Gamma)} \phi(x)$$

e.g. $\phi = \sum$ Shannon entropy

$$= \inf_{p \in \mathcal{P}} \phi(\text{spec } \sqrt{p} \nabla f(p) \sqrt{p})$$

just saw this

Optimization on Entanglement Polytope



$$\min_{x \in \Delta(\Gamma)} \phi(x)$$

e.g. $\phi = \sum$ Shannon entropy

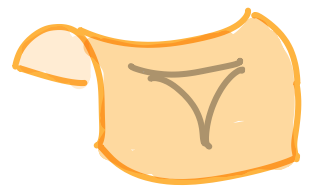
$$= \inf_{p \in \mathcal{P}} \phi(\text{spec } \sqrt{p} \nabla f(p) \sqrt{p})$$

unitarily-invariant lift
(convex if ϕ symmetric)

$$= \inf_{p \in \mathcal{P}} \Phi(\sqrt{p} \nabla f(p) \sqrt{p})$$

e.g. $\Phi = \sum$ quantum entropy

Optimization on Entanglement Polytope



$$\min_{X \in \Delta(\Gamma)} \phi(X)$$

e.g. $\phi = \sum$ Shannon entropy

$$= \inf_{p \in \mathcal{P}} \phi(\text{spec } \sqrt{p} \nabla f(p) \sqrt{p})$$

unitarily-invariant lift

$$= \inf_{p \in \mathcal{P}} \boxed{\Phi(\sqrt{p} \nabla f(p) \sqrt{p})}$$

e.g. $\Phi = \sum$ quantum entropy

What steps to take to minimize? WLOG $p = I$. Then:

$$\nabla(\dots \text{objective} \dots) = \nabla^2 f(I) [\nabla \Phi(\nabla f(I)), \bullet]$$

≥ 0 because
f convex

$\hookrightarrow$ good descent
direction!

cf. Hirai's & talk by Keiya

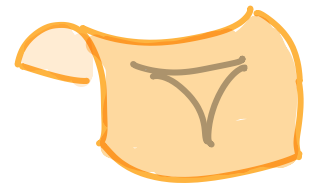
Mirror Descent on Entanglement Polytopes

(and other moment polytopes)

Summary: To minimize convex ϕ over entanglement polytope, lift to unitarily-invariant convex fn. Φ & apply iteration:

$$p \leftarrow \sqrt{p} e^{-\eta \nabla \Phi(\sqrt{p} \nabla f(p) \sqrt{p})} \sqrt{p}$$

suitable step size



Mirror Descent on Entanglement Polytopes

(and other moment polytopes)

Summary: To minimize convex ϕ over entanglement polytope, lift to unitarily-invariant convex fn. Φ & apply iteration:

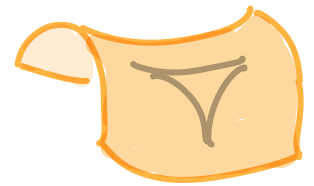
$$p \leftarrow \sqrt{p} e^{-\eta \nabla \Phi(\sqrt{p} \nabla f(p) \sqrt{p})} \sqrt{p}$$

i.e.

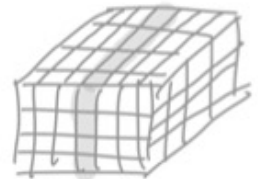
$$\Sigma \leftarrow e^{-\frac{\eta}{2} \nabla \Phi(\sigma_1, \dots, \sigma_d)} \cdot \Sigma$$

substep size

q. marginals



$$\Sigma = \sqrt{p} \cdot T$$



Mirror Descent on Entanglement Polytopes

(and other moment polytopes)

Summary: To minimize convex ϕ over entanglement polytope, lift to unitarily-invariant convex fn. Φ & apply iteration:

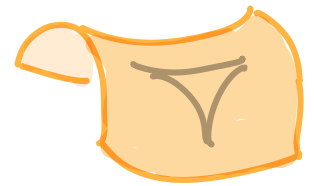
$$p \leftarrow \sqrt{p} e^{-\eta \nabla \Phi(\sqrt{p} \nabla f(p) \sqrt{p})} \sqrt{p}$$

i.e.

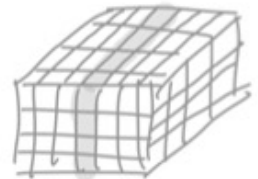
$$\Sigma \leftarrow e^{-\frac{\eta}{2} \nabla \Phi(\sigma_1, \dots, \sigma_d)} \cdot \Sigma$$

suitable step size

q. marginals



$$\Sigma = \sqrt{p} \cdot T$$



easy-to-check criterion:
quantitative convergence
for explicit step size

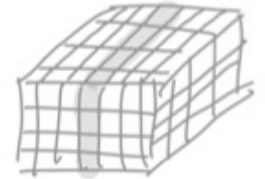
← can be understood as version of "mirror descent" to Hadamard setting

perspective ↓ also helped resolve question by Stollen

Mirror Descent on Entanglement Polytopes

Concretely: To minimize ϕ , run

$$S' \leftarrow e^{-\frac{\eta}{2} \nabla \Phi(\sigma_1, \dots, \sigma_d)} \cdot S$$



Example: $\phi = \frac{1}{2} \|\cdot\|_2^2$

Q. Functionals:

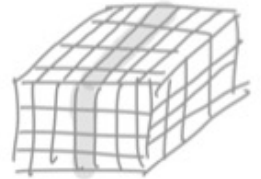
$$\phi = \sum_j \theta_j \cdot H(x_j)$$

Shannon
entropy

Mirror Descent on Entanglement Polytopes

Concretely: To minimize ϕ , run

$$S' \leftarrow e^{-\frac{\eta}{2} \nabla \Phi(\sigma_1, \dots, \sigma_d)} \cdot S$$



Example: $\phi = \frac{1}{2} \|\cdot\|_2^2$

$$\hookrightarrow \Phi = \frac{1}{2} \|\cdot\|_F^2$$

$$\hookrightarrow \nabla \Phi(\sigma_1, \dots, \sigma_d) = (\sigma_1, \dots, \sigma_d)$$

Q. Functionals:

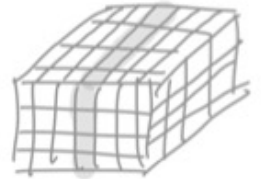
$$\phi = \sum_j \theta_j \cdot H(x_j)$$

Shannon
entropy

Mirror Descent on Entanglement Polytopes

Concretely: To minimize ϕ , run

$$S' \leftarrow e^{-\frac{\eta}{2} \nabla \Phi(\sigma_1, \dots, \sigma_d)} \cdot S$$



Example: $\phi = \frac{1}{2} \|\cdot\|_2^2$

$\hookrightarrow \Phi = \frac{1}{2} \|\cdot\|_F^2$

$\hookrightarrow \nabla \Phi(\sigma_1, \dots, \sigma_d) = (\sigma_1, \dots, \sigma_d)$

$$S \leftarrow (e^{-\frac{\eta}{2} \sigma_1} \otimes \dots \otimes e^{-\frac{\eta}{2} \sigma_d}) S$$

- ordinary gradient descent
- unlike flip-flop, also converges for unstable T

Q. Functionals:

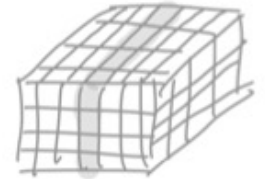
$$\phi = \sum_j \theta_j \cdot H(x_j)$$

Shannon
entropy

Mirror Descent on Entanglement Polytopes

Concretely: To minimize ϕ , run

$$S' \leftarrow e^{-\frac{\eta}{2} \nabla \Phi(\sigma_1, \dots, \sigma_d)} \cdot S$$



Example: $\phi = \frac{1}{2} \|\cdot\|_2^2$

$$\hookrightarrow \Phi = \frac{1}{2} \|\cdot\|_F^2$$

$$\hookrightarrow \nabla \Phi(\sigma_1, \dots, \sigma_d) = (\sigma_1, \dots, \sigma_d)$$

$$S \leftarrow (e^{-\frac{\eta}{2} \sigma_1} \otimes \dots \otimes e^{-\frac{\eta}{2} \sigma_d}) S$$

- ordinary gradient descent
- unlike flip-flop, also converges for unstable T

Q. Functionals:

$$\phi = \sum_j \theta_j \cdot H(\kappa_j)$$

Shannon entropy

$$\hookrightarrow \Phi = \sum_j \theta_j \cdot H(\sigma_j)$$

quantum entropy

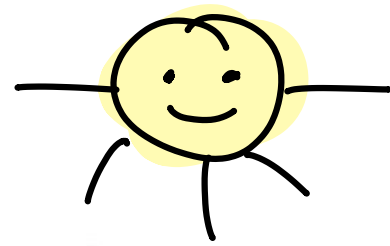
$$\hookrightarrow \nabla \Phi(\sigma_1, \dots, \sigma_d) \approx (\theta_1 \cdot \log \sigma_1, \dots, \theta_d \cdot \log \sigma_d)$$

$$S \leftarrow (\sigma_1^{-\frac{\theta_1}{2}} \otimes \dots \otimes \sigma_d^{-\frac{\theta_d}{2}}) S$$

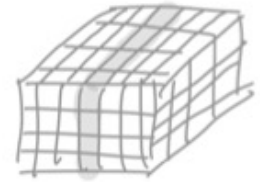
Phew ☺

19/20

Summary & Outlook

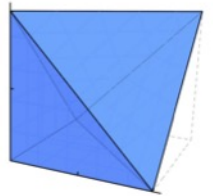
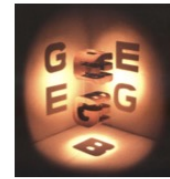


Tensors appear in many different roles

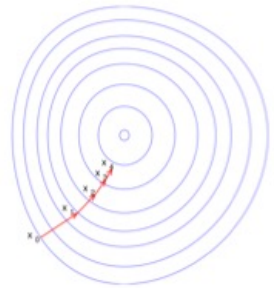


q. information, algebraic problems, statistical samples...

Symmetries / group actions create links
and give interesting problems & quantities..



...that can be tackled effectively using **convex optimization**
on curved spaces



Thanks for Your Attention !

...along with some
algebraic geometry
invariant theory
∞
😊